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G_p-metric spaces—symmetric and asymmetric

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Abstract: In this paper, we discuss some results in the framework of G_p-metric spaces, established recently in several papers. The main purpose is to complement and explain the theoretical approach in the development of G_p-metric spaces. Some examples are given to support our theoretical conclusions.

Keywords: G_p-metric space, complete G_p-metric space, symmetric G_p-metric space, asymmetric G_p-metric space.

1 Introduction and preliminaries

Partial and G-metric spaces are two important kinds of generalized metric spaces. Partial metric spaces were introduced by Matthews [14] in 1994 as follows:

Definition 1.1. Let X be a nonempty set. A partial metric or p-metric is a mapping $p : X^2 \to [0,+\infty)$ which satisfies:
(p1) for all $x,y \in X$, $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$;
(p2) $p(x,x) \le p(x,y)$, for all $x,y \in X$;
(p3) $p(x,y) = p(y,x)$, for all $x,y \in X$;
(p4) $p(x,z) \le p(x,y) + p(y,z) - p(y,y)$, for all $x,y,z \in X$.
The pair (X,p) is called a partial metric space.

It is clear that each (standard) metric space is a partial metric space, while the converse is not true in general. Many authors have obtained different kinds of results in partial metric spaces, for example, fixed point theorems for operators satisfying various contractive conditions (see, e.g., [10, 14, 16, 17, 21] as well as the references therein).

On the other hand, in 2006, Mustafa and Sims [15] introduced still another new kind of generalized metric spaces, named as G-metric space as follows:

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Definition 1.2. Let X be a nonempty set. A generalized metric or G-metric is a mapping $G: X^3 \to [0,+\infty)$ which satisfies the following properties:

(G1) for all $x,y,z \in X$, $x = y = z \Leftrightarrow G(x,y,z) = 0$;

(G2) $0 < G(x,x,y)$, for all $x,y \in X$ with $x \neq y$;

(G3) $G(x,x,y) \leq G(x,y,z)$, for all $x,y,z \in$ with $z \neq y$;

(G4) $G(x,y,z) = G(P\{x,y,z\})$, where P is any permutation of x,y,z (symmetry in all three variables);

(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$, for all $x,y,z,a \in X$ (rectangle inequality).

The pair (X,G) is called a G-metric space.

Based on this notion, many fixed point results under different contractive conditions have been obtained (see, e.g., [1, 8, 9, 15], as well as the references therein). It should be noted that results of this kind are much easier to obtain in the symmetric case, i.e., when $G(x,x,y) = G(x,y,y)$ holds for all $x,y \in X$ (see also [11]).

In 2011, Ahmadi Zand and Nezhad [2] attempted to introduce a new generalization of both partial metric spaces and G-metric spaces, by defining the notion of G_p-metric space in the following manner.

Definition 1.3. Let X be a nonempty set. A mapping $G_p : X^3 \to [0,+\infty)$ is called a G_p-metric if the following conditions are satisfied:

(G_p1) for $x,y,z \in X$, if $G_p(x,y,z) = G_p(z,z,z) = G_p(y,y,y) = G_p(x,x,x)$ then $x = y = z$;

(G_p2) $G_p(x,x,x) \leq G_p(x,x,y) \leq G_p(x,y,z)$, for all $x,y,z \in X$;

(G_p3) $G_p(x,y,z) = G_p(P\{x,y,z\})$, where P is any permutation of x,y,z (symmetry in all three variables);

(G_p4) $G_p(x,y,z) \leq G_p(x,a,a) + G_p(a,y,z) - G_p(a,a,a)$, for all $x,y,z,a \in X$ (rectangle inequality).

The pair (X,G_p) is called a G_p-metric space.

Following this definition, the authors in [3, 4, 6, 19] obtained several fixed point results in G_p-metric spaces. However, it is clear that the assumption (G_p2) of the previous definition readily implies that

$$G_p(x,x,y) = G_p(x,y,y), \text{ for all } x,y \in X. \tag{1}$$

Hence, the claim in [2] (page 87, lines 6^-, 7^-) that each G-metric space is also a G_p-metric space is false, since it is well known that (1) might not hold in a G-metric space. Also, Definition 6 in [2] is superfluous since all G_p-metric spaces (in the sense of Definition 1.3) are symmetric.

In order to overcome this problem, the authors introduced in [18] another definition of G_p-metric spaces as follows.

Definition 1.4. Let X be a nonempty set. A mapping $G_p : X^3 \to [0,+\infty)$ is called a G_p-metric if the conditions (G_p1), (G_p3) and (G_p4) of Definition 1.3 are satisfied, while the condition (G_p2) is replaced by

(G_p2') $G_p(x,x,x) \leq G_p(x,x,y) \leq G_p(x,y,z)$ for all $x,y,z \in X$ with $z \neq y$.

They showed that in this case each G-metric space is a G_p-metric space, but a G_p-metric space might be *asymmetric*, i.e., the condition (1) might not hold, as the following example shows.

Example 1.5. [18] Let $X = \{0,1,2,3\}$ and let

$$\begin{aligned} A = \{&(1,0,0),(0,1,0),(0,0,1),(2,0,0),(0,2,0),(0,0,2),(3,0,0),(0,3,0),(0,0,3),\\ &(1,2,2),(2,1,2),(2,2,1),(1,3,3),(3,1,3),(3,3,1),(2,3,3),(3,2,3),(3,3,2)\},\\ B = \{&(0,1,1),(1,0,1),(1,1,0),(0,2,2),(2,0,2),(2,2,0),(0,3,3),(3,0,3),(3,3,0)\\ &(2,1,1),(1,2,1),(1,1,2),(3,1,1),(1,3,1),(1,1,3),(3,2,2),(2,3,2),(2,2,3)\} \end{aligned}$$

(note that, in [18], the terms $(1,3,3),(3,1,3),(3,3,1)$, and $(3,1,1),(1,3,1),(1,1,3)$ were missing in the sets A and B, respectively). Define $G_p : X^3 \to \mathbb{R}^+$ by

$$G(x,y,z) = \begin{cases} 1, & \text{if } x = y = z \neq 2,\\ 0, & \text{if } x = y = z = 2,\\ 2, & \text{if } (x,y,z) \in A,\\ \frac{5}{2}, & \text{if } (x,y,z) \in B,\\ 3, & \text{if } x \neq y \neq z \neq x. \end{cases}$$

It is easy to check that (X,G_p) is an asymmetric G_p-metric space.

Using Definition 1.4, some structural and fixed point results were obtained in [7, 18, 20]. However, the proofs of some of these results used auxiliary assertions taken from [2] which were deduced under the assumption that (G_p2) holds. Hence, such results are under question and have to checked.

The purpose of this paper is to clarify the terminology on G_p-spaces and provide (with proofs) exact formulations of certain structural properties of such spaces.

In what follows, in order to avoid confusion, the spaces satisfying the conditions of Definition 1.4 will be called *G_p-metric spaces*, while the spaces satisfying the conditions of Definition 1.3 will be called *symmetric G_p-metric spaces*.

2 Structural results

First note that, putting $x = y$ and $a = z$ in the inequality (G_p4), we obtain that

$$G_p(x,x,z) \leq 2G_p(x,z,z) - G_p(z,z,z) \tag{2}$$

holds for all $x,z \in X$ (both in symmetric and asymmetric cases).

As our first result, we provide the following simple generalization of [3, Lemma 1.10] (see also [4, 6, 7, 18, 20]).

Proposition 2.1. Let (X,G_p) be a G_p-metric space. Then

1° if, for some $x,y,z\in X$, $G_p(x,y,z)=0$, then $x=y=z$;
2° if $x\neq y$, then $G_p(x,y,y)>0$.

Proof. 1° In the case that $x\neq y\neq z\neq x$, the statement 1° is an immediate consequence of (G_p2') and (G_p1). If, for instance, $x\neq y=z$ then (G_p2') implies that $0=G_p(x,y,z)\geq G_p(z,z,x)\geq G_p(z,z,z)=G_p(y,y,y)$, so that $G_p(y,y,y)=G_p(x,y,y)=0$. On the other hand, (2) implies that $G_p(x,x,y)\leq 2G_p(x,y,y)-G_p(y,y,y)=2\cdot 0-0=0$. In a similar manner we can obtain that $G_p(x,x,x)=G_p(x,x,y)=G_p(y,y,y)=0$. Now we have that

$$G_p(x,y,y)=G_p(x,x,y)=G_p(x,x,x)=G_p(y,y,y),$$

wherefrom, by (G_p1) the result follows.

2° Let $G_p(x,y,y)=0$. Now, as in the proof for 1° when $x\neq y=z$ we obtain that $x=y$. A contradiction. □

Proposition 2.2. To every G_p-metric space (X,G_p) there corresponds a metric space $\left(X,d_{G_p}\right)$ with d_{G_p} defined by

$$d_{G_p}(x,y)=G_p(x,y,y)+G_p(x,x,y)-G_p(x,x,x)-G_p(y,y,y),\ \text{for all } x,y\in X. \quad (3)$$

Proof. Since, by (G_p2'), $G_p(x,x,x)\leq G_p(x,x,y)$ for all $x,y\in X$ we have that $d_{G_p}(x,y)\geq 0$ for all $x,y\in X$, Also, if $x=y$ then $d_{G_p}(x,y)=0$. Conversely, let $d_{G_p}(x,y)=0$. In this case we have that

$$G_p(x,y,y)+G_p(x,x,y)-G_p(x,x,x)-G_p(y,y,y)=0,$$

that is,

$$[G_p(x,x,y)-G_p(x,x,x)]+[G_p(x,y,y)-G_p(y,y,y)]=0,$$

or, equivalently, $G_p(x,x,y)=G_p(x,x,x)$ and $G_p(x,y,y)=G_p(y,y,y)$. Further, (G_p4) implies that $G_p(x,y,y)\leq 2G_p(x,x,y)-G_p(x,x,x)=G_p(x,x,y)$ and similarly $G_p(x,x,y)\leq G_p(x,y,y)$ for the given $x,y\in X$. We finally obtain that

$$G_p(x,y,x)=G_p(x,x,x)=G_p(y,y,y),$$

which according to (G_p1) gives us $x=y$.

It is obvious that $d_{G_p}(x,y)=d_{G_p}(y,x)$ for all $x,y\in X$.

Finally, we shall prove that

$$d_{G_p}(x,z)\leq d_{G_p}(x,y)+d_{G_p}(y,z),$$

for all $x,y,z\in X$ or, equivalently,

$$\begin{aligned}G_p(x,x,z)+G_p(x,z,z)-G_p(x,x,x)-G_p(z,z,z)&\\ \leq G_p(x,x,y)+G_p(x,y,y)-G_p(x,x,x)-G_p(y,y,y)&\\ +G_p(y,y,z)+G_p(y,z,z)-G_p(y,y,y)-G_p(z,z,z),&\end{aligned}$$

that is,

$$G_p(x,x,z)+G_p(x,z,z)$$
$$\le G_p(x,x,y)+G_p(x,y,y)-G_p(y,y,y)+G_p(y,y,z)+G_p(y,z,z)-G_p(y,y,y).$$

However,

$$G_p(x,x,z)=G_p(z,x,x)\le G_p(z,y,y)+G_p(y,x,x)-G_p(y,y,y)$$
$$\text{and } G_p(x,z,z)\le G_p(x,y,y)+G_p(y,z,z)-G_p(y,y,y),$$

which gives us the result. □

Remark 2.3. The proof of the previous assertion in [2, Proposition 2] is not complete. In [18, Proposition 1.7], and in most of the other mentioned papers, it is given without proof.

Example 2.4. Let $X=\{a,b\}$ be equipped with G_p-metric defined as

$$G_p(a,a,a)=0, \qquad G_p(a,a,b)=G_p(a,b,a)=G_p(b,a,a)=1 \text{ and}$$
$$G_p(b,b,b)=G_p(a,b,b)=G_p(b,a,b)=G_p(b,b,a)=2.$$

Since $G_p(a,a,b)\neq G_p(a,b,b)$ we get that (X,G_p) is an asymmetric G_p-metric space. Also, we have that for all $x,y\in X$:

$$d_{G_p}(x,y)=G_p(x,x,y)+G_p(x,y,y)-G_p(x,x,x)-G_p(y,y,y)=\begin{cases}1, & x\neq y\\ 0, & x=y.\end{cases}$$

is a (standard) metric on X. □

Definition 2.5. [18] Let (X,G_p) be a G_p-metric space and let $\{x_n\}$ be a sequence of points in X. A point $x\in X$ is said to be a limit of the sequence $\{x_n\}_{n\in\mathbb{N}}$, denoted by $x_n\to x$, if $\lim_{n,m\to\infty}G_p(x,x_n,x_m)=G_p(x,x,x)$.

The following is easy to show (see also [2, Proposition 4]).

Proposition 2.6. Let (X,G_p) be a symmetric G_p-metric space. Then for a sequence $\{x_n\}\subseteq X$ and a point $x\in X$ the following are equivalent:
(1) $\{x_n\}$ is G_p-convergent to x;
(2) $G_p(x_n,x_n,x)\to G_p(x,x,x)$, as $n\to\infty$;
(3) $G_p(x_n,x,x)\to G_p(x,x,x)$, as $n\to\infty$.

Proof. Since (X,G_p) is a symmetric G_p-metric space then (2) $\Leftrightarrow$ (3). Taking in (1) $m=n$ we have that (1) implies (2). For the converse we have:

$$G_p(x,x_n,x_m)-G_p(x,x,x)$$
$$=G_p(x_n,x_m,x)-G_p(x,x,x) \text{ (by (}G_p4\text{) with } a=x \text{ we have)}$$
$$\le G_p(x_n,x,x)+G_p(x,x_m,x)-G_p(x,x,x)-G_p(x,x,x)$$
$$=[G_p(x_n,x,x)-G_p(x,x,x)]+[G_p(x_m,x,x)-G_p(x,x,x)]$$
$$\to 0+0=0, \text{ as } n,m\to\infty,$$

i.e. (3) implies (1). The proof is complete. □

Remark 2.7. In the asymmetric case, Proposition 2.6. does not hold, as is shown by the following example. In other words, [18, Proposition 1.10], and the respective assertion in most of the other mentioned papers, are not true in this case.

Example 2.8. Let $X = \{a,b\}$ be equipped with the G_p-metric defined as

$$G_p(a,a,a) = 0, \qquad G_p(a,a,b) = G_p(a,b,a) = G_p(b,a,a) = 1 = G_p(b,b,b)$$
$$\text{and } G_p(a,b,b) = G_p(b,a,b) = G_p(b,b,a) = 2.$$

In this example we have that the sequence $x_n = a$ for all $n \in \mathbb{N}$ converges to a as well as to b. However, the conditions (2) and (3) of Proposition 2.5 are not equivalent. Indeed,

$$G_p(x_n,x_n,b) = G_p(a,a,b) \to 1 = G_p(b,b,b),$$

while

$$G_p(x_n,b,b) = G_p(a,b,b) \to 2 \neq G_p(b,b,b),$$

as $n \to \infty$.

This example also shows that the function $G_p(\cdot,\cdot,\cdot)$ need not be continuous in the sense that $x_n \to x, y_n \to y$ and $z_n \to z$ imply $G_p(x_n,y_n,z_n) \to G_p(x,y,z)$. Indeed, we can take $x_n = y_n = a$ and $z_n = b$ for all $n \in \mathbb{N}$. Then, it is easy to check that $x_n \to b$, $y_n \to a$ and $z_n \to b$ but $G_p(x_n,y_n,z_n) \nrightarrow G_p(b,a,b)$, because $G_p(x_n,y_n,z_n) = G_p(a,a,b) = 1 \neq 2 = G_p(b,a,b)$. □

Definition of a G_p-Cauchy sequence *does not* appear in the paper [2], although the following definition is cited in most of the other mentioned articles as taken from [2].

Definition 2.9. 1. The sequence $\{x_n\}_{n\in\mathbb{N}}$ in a G_p-metric space (X,G_p) is said to be a G_p-Cauchy sequence if there exists $r \in \mathbb{R}$ such that $\lim_{n,m\to\infty} G_p(x_n,x_m,x_m) = r$.

2. (X,G_p) is said to be G_p-complete if for every G_p-Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$ there exists $x \in X$ such that

$$\lim_{n,m\to\infty} G_p(x_n,x_m,x_m) = \lim_{n,m\to\infty} G_p(x_n,x_m,x) = G_p(x,x,x).$$

Proposition 2.10. Let (X,G_p) be a G_p-metric space and $\{x_n\}$ be a sequence in X.

1. $\lim_{n,m\to\infty} G_p(x_n,x_m,x_m) = r$ if and only if $\lim_{n,m\to\infty} G_p(x_n,x_n,x_m) = r$.

2. If the space (X,G_p) is symmetric then $\{x_n\}$ is a G_p-Cauchy sequence if and only if $\lim_{l,m,n\to\infty} G_p(x_l,x_m,x_n) = r$ for some $r \in \mathbb{R}$.

Proof. **1.** Suppose that $\lim_{n,m\to\infty} G_p(x_n,x_m,x_m) = r$, i.e., for each $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $|G_p(x_n,x_m,x_m) - r| < \varepsilon$ for all $m,n \geq n_0$. Then, also $|G_p(x_m,x_n,x_n) - r| < \varepsilon$ for $n,m \geq n_0$ (even in the asymmetric case) and hence $\lim_{n,m\to\infty} G_p(x_n,x_n,x_m) = r$.

2. Let (X,G_p) be a symmetric G_p-space. By (G_p4) we have

$$G_p(x_l,x_n,x_m) \leq G_p(x_l,x_n,x_n) + G_p(x_n,x_n,x_m) - G(x_n,x_n,x_n),$$

i.e.,

$$G_p(x_l,x_n,x_m) - G_p(x_n,x_n,x_m) \leq G_p(x_l,x_n,x_n) - G_p(x_n,x_n,x_n).$$

Further, by (G_p2), $G_p(x_n,x_n,x_m) \le G_p(x_n,x_m,x_l)$. Hence, we have

$$0 \le G_p(x_l,x_n,x_m) - G_p(x_n,x_n,x_m) \le G_p(x_l,x_n,x_n) - G_p(x_n,x_n,x_n).$$

Since, by the assumption, the right-hand side of the previous inequality tends to 0 as $l,n \to \infty$, we get that $G_p(x_l,x_n,x_m) \to r$ as $l,m,n \to \infty$. The converse is obvious. □

Proposition 2.11. A sequence $\{x_n\}$ in a G_p-metric space (X,G_p) is a G_p-Cauchy sequence if and only if it is a d_{G_p}-Cauchy sequence.

Proof. Suppose that $\{x_n\}$ is a G_p-Cauchy sequence, i.e., $\lim_{m,n\to\infty} G_p(x_m,x_n,x_n) = \lim_{m,n\to\infty} G_p(x_n,x_m,x_m) = r$ for some $r \in \mathbb{R}$. From Proposition 2.10 and the definition of metric d_{G_p} we get

$$\begin{aligned}
|d_{G_p}(x_n,x_m)| &= |G_p(x_n,x_m,x_m) + G_p(x_n,x_n,x_m) - G_p(x_n,x_n,x_n) - G_p(x_m,x_m,x_m)| \\
&\le |G_p(x_n,x_m,x_m - r| + |G_p(x_n,x_n,x_m) - r| + |r - G_p(x_n,x_n,x_n)| \\
&\quad + |r - G_p(x_m,x_m,x_m)| \\
&\to 0+0+0+0 = 0, \text{ as } m,n \to \infty.
\end{aligned}$$

Conversely, let

$$G_p(x_n,x_m,x_m) - G_p(x_m,x_m,x_m) + G_p(x_n,x_n,x_m) - G_p(x_n,x_n,x_n) \to 0,$$

as $m,n \to \infty$. By (G_p2'), this implies that

$$G_p(x_n,x_m,x_m) - G_p(x_m,x_m,x_m) \to 0 \text{ and } G_p(x_n,x_n,x_m) - G_p(x_n,x_n,x_n) \to 0, \tag{4}$$

as $m,n \to \infty$. Suppose, without loss of generality, that $G_p(x_n,x_n,x_n) \ge G_p(x_m,x_m,x_m)$. Then, using ($G_p2'$) and (2), we get that

$$\begin{aligned}
0 &\le G_p(x_n,x_n,x_n) - G_p(x_m,x_m,x_m) \le G_p(x_n,x_n,x_m) - G_p(x_m,x_m,x_m) \\
&\le 2G_p(x_n,x_m,x_m) - G_p(x_m,x_m,x_m) - G_p(x_m,x_m,x_m) \\
&= 2(G_p(x_n,x_m,x_m) - G_p(x_m,x_m,x_m)).
\end{aligned}$$

Applying (4), we see that $|G_p(x_n,x_n,x_n) - G_p(x_m,x_m,x_m)| \to 0$ as $m,n \to \infty$. Thus, $\{G_p(x_n,x_n,x_n)\}$ is a Cauchy sequence of real numbers, converging to some $r \in \mathbb{R}$ as $n \to \infty$. Then we have

$$\begin{aligned}
|G_p(x_n,x_m,x_m) - r| &\le |G_p(x_n,x_m,x_m) - G_p(x_m,x_m,x_m)| + |G_p(x_m,x_m,x_m) - r| \\
&\to 0+0 = 0 \text{ as } m,n \to \infty.
\end{aligned}$$

Hence, $\{x_n\}$ is a G_p-Cauchy sequence. □

Remark 2.12. This result was given as [18, Lemma 1.12.(1)], but the proof implicitly used the symmetry of the space.

Proposition 2.13. A G_p-metric space (X,G_p) is complete if and only if the metric space (X,d_{G_p}) is complete.

Proof. Suppose that (X, d_{G_p}) is complete, and let $\{x_n\}$ be a G_p-Cauchy sequence in (X, G_p). By Proposition 2.11, it is also a d_{G_p}-Cauchy sequence, hence converging to some $x \in X$. It means that $d_{G_p}(x_n, x) \to 0$ as $n \to \infty$, i.e.,

$$[G_p(x_n,x_n,x) - G_p(x_n,x_n,x_n)] + [G_p(x_n,x,x) - G_p(x,x,x)] \to 0,$$

as $n \to \infty$. Then, applying (G_p4),

$$\begin{aligned} G_p(x_n,x_m,x) - G_p(x,x,x) &\le G_p(x_n,x,x) + G_p(x,x_m,x) - G_p(x,x,x) - G_p(x,x,x) \\ &= [G_p(x_n,x,x) - G_p(x,x,x)] + [G_p(x_m,x,x) - G_p(x,x,x)] \\ &\to 0 + 0 = 0, \end{aligned}$$

as $m, n \to \infty$. Thus, $x_n \to x$ in (X, G_p) and (X, G_p) is G_p-complete.

Conversely, let the (X, G_p) be a G_p-complete space, and let $\{x_n\}$ be a d_{G_p}-Cauchy sequence in X. By Proposition 2.11, it is also a G_p-Cauchy sequence. It follows that there exists $x \in X$ such that

$$\lim_{n,m\to\infty} G_p(x_n,x_m,x_m) = \lim_{n,m\to\infty} G_p(x_n,x_m,x) = G_p(x,x,x).$$

In particular, $\lim_{n\to\infty} G_p(x_n,x_n,x) = \lim_{n\to\infty} G_p(x_n,x_n,x_n) = G_p(x,x,x)$.

On the other hand, by (G_p2') and (2) we get

$$G_p(x,x,x) \le G_p(x,x,x_n) \le 2G_p(x,x_n,x_n) - G(x_n,x_n,x_n)$$

and this implies that also $\lim_{n\to\infty} G_p(x_n,x,x) = G_p(x,x,x)$. Hence,

$$[G_p(x_n,x_n,x) - G_p(x_n,x_n,x_n)] + [G_p(x_n,x,x) - G_p(x,x,x)] \to 0,$$

as $n \to \infty$. Thus, $d_{G_p}(x_n, x) \to 0$ and the space (X, d_{G_p}) is complete. □

Remark 2.14. Similar as Remark 2.12.

3 Comments on some fixed point results

As already stated, several fixed point results in G_p-metric spaces were presented in the papers [3, 4, 6, 7, 12, 13, 18, 20]. Nearly all of these results were obtained in symmetric G_p-spaces, i.e., using axiom (G_p2), or using, explicitly or implicitly, some of the results from paper [2] that depend on this axiom. Hence, it is an open question whether these results are true in arbitrary G_p-metric spaces, i.e., whether they can be obtained using axiom (G_p2') instead of (G_p2).

Note also that nearly all of the examples presented in the mentioned articles are given in symmetric G_p-metric spaces (in most of them just $G_p(x,y,z) = \max\{x,y,z\}$ on $\mathbb{R}^+$), which is a much less interesting situation. Namely, it is well-known that already in the class of

G-metric spaces, symmetric spaces are not so interesting since the respective results can be easily reduced to the standard metric ones (see, e.g., [11, 15]).

We mention here some situations of this kind, besides those stated earlier in this text, noting that similar remarks can be applied to most of the results of the mentioned articles.

In the paper [3], Lemma 2.9 is true only in the symmetric case and is false in the asymmetric one (see our Example 2.8). Hence, Theorem 2.10 of this paper is under question, since it uses Lemma 2.9 (see relation (2.58) on page 13).

Similarly, in the paper [7], the proof of Theorem 3.3 uses the relation (48) on page 18, which is wrong in the asymmetric case. The same holds for the proof of Theorem 3.1 of the paper [12] (see line 10 of page 9), and also for [19, Theorem 2.4] (see the end of the proof).

In the paper [18], the proof of Theorem 2.2 uses (on page 89) Lemma 1.13 of that article, the proof of which depends on symmetry of the space. The same applies to the proof of [20, Theorem 3.1].

4 Conclusion

There are two possible definitions of G_p-metric spaces in the literature—the one introduced in [2] uses a stronger assumption which implies a symmetry property, while the one used in [18] enables the consideration of a wider class of examples. In the first case, it is much easier to obtain several fixed point results, but these results are rather weak since the corresponding class of spaces does not even contain G-metric spaces (which may be asymmetric). The other definition is more natural, however, fixed point results are much harder to obtain.

In this article, several propositions are presented, explaining precise relationship between structural properties of G_p-metric spaces in the symmetric and asymmetric cases. Also, some notes on validity of fixed point results in several papers are given.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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