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Second Order Statistic Analysis of Selection Macro-Diversity Combining over Gama Shadowed Rayleigh Fading Channels

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Abstract: In this paper an approach to the second order statistics analysis of macrodiversity system operating over the Gamma shadowed Rayleigh fading channels is presented. Simultaneous influence of multipath fading and shadowing is allievated through the usage of macrodiversity system. We have considered SC (selection combining) macrodiversity system consisting of two base stations (microdiversity systems). Cases of MRC (Maximal Ratio Combining) and SC diversity with arbitrary number of branches, applied at microlevel over the Rayleigh fading channels are discussed. Selection between maco-combiners is based on output signal power values. Numerical results for LCR (Level crossing rate) at the output of this system are presented and discussed in the function of various system parameters.

Keywords: Macrodiversity system, Markov chains, Gamma distribution

1 Introduction

Multipath fading and shadowing conditions should be simultaneously taken into account, since they both coexist in wireless systems [1].

The short-term signal variation (multipath fading) can be described by several distributions such as Hoyt, Rayleigh, Rice, Nakagami-*m*, and Weibull. Rayleigh fading describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. In provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications.

The long-term signal variation (shadowing) is often described by lognormal and Gamma distribution. In cellular networks, long-term fading can put a heavy limit on system performance. Shadowing is the result of the topographical elements and other structures in the transmission path such as trees, tall buildings [2]. We must simultaneously take short- and long-term fading conditions into account since they both coexist in wireless systems.

Various techniques for reducing short-term fading effect are used in wireless communication systems [3]. Upgrading transmission reliability without increasing transmission

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power and bandwidth while increasing channel capacity is the main goal of diversity techniques. Diversity reception is an effective remedy that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal. An efficient method for amelioration system's quality of service (QoS) with using multiple receiver antennas is called space diversity. Several principal types of combining techniques can be generally performed by their dependence on complexity restriction put on the communication system and amount of channel state information available at the receiver. Combining techniques like maximal ratio combining (MRC) and equal gain combining (EGC) and require all or some of the amount of the channel state information of received signal. Second, MRC and EGC require separate receiver chain for each branch of the diversity system, which increase it complexity of system. One of the least complicated combining methods is selection combining (SC). In opposition to previous combining techniques, SC receiver processes only one of the diversity branches, and is much simpler for practical realization. Generally, selection combining, selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [1, 3-4], assuming that noise power is equally distributed over branches.

While short-term fading is mitigated through the use of diversity techniques typically at the single base station (micro-diversity), use of such microdiversity approaches alone will not be sufficient to mitigate the overall channel degradation when shadowing is also concurrently present. Macro-diversity is used to alleviate the effects of shadowing, where multiple signals are received at widely located radio ports, ensuring that different long-term fading is experienced by these signals [5].

The level crossing rate (LCR) and the average fading duration (AFD) are second-order statistical quantities, which complement the static probabilistic description of the fading signal (the first-order statistics), and have found several applications in the modelling and design of practical systems and designing wireless communication systems. Actually, these second-order statistical measures are related to criterion used to assess error probability of packets of distinct length and to determinate parameters of equivalent channel, modeled by a Markov chain with defined number of states [6].

In this paper LCR and AFD values at the output of the macrodiversity system operating over the Gamma shadowed Rayleigh fading channels are determined for the cases when SC and MRC combining techniques are applied at the microlevel.

2 System model

Let $z$be the received signal envelope, and $\dot{z}$ its derivative with respect to time, with joined probability density function (PDF) $p_{z\,\dot{z}}(z\dot{z})$.The level crossing rate (LCR) at the envelope $z$is defined as the rate at which a fading signal envelope crosses level $z$in a positive or a negative direction and is mathematically defined by formula [6]:

$$N_Z(Z) = \int_0^\infty \dot{Z} p_{Z\dot{Z}}\left(Z, \dot{Z}\right) d\dot{Z} \tag{1}$$

The average fade duration (AFD) is defined as the average time over which the signal envelope ratio remains below a specified level after crossing that level in a downward direction, and is determinated as [6]:

$$T_Z(Z) = \frac{F_z(z \leq Z)}{N_Z(Z)} \tag{2}$$

Macrodiversity combiner is SC type combiner, which consists of two microdiversity systems and selection is based on their output signal's average power values. Output statistics can then be presented in the form.

$$\begin{aligned} p_{z,\dot{z}}(z,\dot{z}) &= \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 p_{z_1,\dot{z}_1}\left(z,\dot{z}/\Omega_1\right) p_{\Omega_1,\dot{\Omega}_1}(\Omega_1\Omega_2) + \\ &+ \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 p_{z_2,\dot{z}_2}\left(z,\dot{z}/\Omega_2\right) p_{\Omega_1,\dot{\Omega}_1}(\Omega_1\Omega_2) \end{aligned} \tag{3}$$

$$\begin{aligned} F_z(z) = \int_0^\infty d\Omega_1 \int_0^{\Omega_1} d\Omega_2 F_{z_1}(z/\Omega_1) P_{\Omega_1\Omega_2}(\Omega_1\Omega_2) + \\ + \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 F_{z_2}(z/\Omega_2) P_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \end{aligned} \tag{4}$$

Here Ω_1 and Ω_2 denote average output powers of signals of microcombiners and $p(\Omega_1, \Omega_2)$ represents their joint probability density function modeled by joint Gamma distribution.

2.1 Selection combining at micro-level

The joint probability density functions at the output of the microcombiners, since a regular selection combining is performed, can be written as:

$$p_{z_i,\dot{z}_i}(z_i,\dot{z}_i/\Omega_i) = \sum_{j=1}^{N_i} p_{z_{ij}}(z_{ij}/\Omega_i) p_{\dot{z}_{ij}}(\dot{z}_{ij}) \prod_{\substack{k=1 \\ k \neq j}}^{N_i} F_{z_{ik}}(z_{ik}/\Omega_i) \tag{5}$$

with i =1,2 denoting microdiversity system and $j = 1,2,\ldots N_i$number of diversity branches in each of microcombiners. Probability density functions and cumulative distribution functions of Rayleigh short-term (multipath) fading processes at the inputs of microdiversity combiners are given with:

$$p_{z_{ij}}(z_{ij},/\Omega_i) = \frac{2z_{ij}}{\Omega_i} \exp\left(-\frac{z_{ij}^2}{\Omega_i}\right)$$

$$F_{z_{ij}}\left(z_{ij},/\Omega_i\right)=\int\limits_0^{z_{ij}} p_{x_{ij}}\left(x_{ij},/\Omega_i\right)dx_{ij}=1-\exp\left(-\frac{z_{ij}^2}{\Omega_i}\right) \quad (6)$$

The probability density function of derivates $\dot{z}$ of the received signals z at the output of both microdiversity systems, with respect to time, are Gaussian PDFs:

$$p\left(\dot{z}_i\right)=\frac{1}{\sqrt{2\pi}\dot{\sigma}_{z_i}}\exp\left(\frac{\dot{z}_i^2}{2\dot{\sigma}_{z_i}^2}\right),\qquad i=1,2 \quad (7)$$

where $\dot{z}_i$ presents derivatives of received microdiversity envelopes and can be expressed as:

$$\dot{z}_i=\max\left\{\dot{z}_{ij},\quad j=1,..,N_i\right\},\qquad i=1,2 \quad (8)$$

For isotropic scattering, $\dot{z}$ is a Gaussian distributed random variable with zero mean and variance can be expressed as [7]:

$$\dot{\sigma}_{z_i}^2=\pi^2 f_d^2\Omega_i^2,\qquad i=1,2 \quad (9)$$

where f_d is a Doppler shift frequency.

There is shadowing at the input of microdiversity systems as well as at the input of macrodiversity system. The slow fading is statistically independent due to sufficient input antenna spacing. In this case we achieve the highest decrease of the fading influence on system's performances The slow fading is modeled by joint Gamma PDF [4]:

$$\begin{aligned} p_{\Omega_1,\Omega_2}\left(\Omega_1,\Omega_2\right) &= p_{\Omega_1}\left(\Omega_1\right)p_{\Omega_2}\left(\Omega_2\right)\\ &= \frac{1}{\Gamma\left(c_1\right)}\frac{\Omega_1^{c_1-1}}{\Omega_{01}^{c_1}}\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\frac{1}{\Gamma\left(c_2\right)}\frac{\Omega_2^{c_2-1}}{\Omega_{02}^{c_2}}\exp\left(-\frac{\Omega_2}{\Omega_{02}}\right) \end{aligned} \quad (10)$$

where Ω_{0i} present mean values at the inputs of macrocombiner, while c_1 and c_2 present orders of Gamma distributions, and they determine measures of the shadowing present in the channel.

After substituting (6), (10) and (3) in (1), we obtain following expressions for normalized level crossing rate, LCR:

$$\begin{aligned} \frac{N_z\left(z\right)}{f_d} &= \left[\int\limits_0^{\infty} H_1\gamma\left(c_2,\frac{\Omega_1}{\Omega_{02}}\right)\cdot\Omega_1^{c_1-1}\cdot\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\dot{\sigma}_{z1}^2\,d\Omega_1+\right.\\ &+ \left.\int\limits_0^{\infty} H_2\gamma\left(c_1,\frac{\Omega_2}{\Omega_{01}}\right)\cdot\Omega_2^{c_2-1}\cdot\exp\left(-\frac{\Omega_2}{\Omega_{02}}\right)\dot{\sigma}_{z2}^2\,d\Omega_2\right] \end{aligned} \quad (11)$$

where $\gamma\left(m,\mathrm{x}\right)$ represents the lower incomplete Gamma function, with:

$$H_1=\frac{1}{\Gamma\left(c_1\right)\Omega_{01}^{c_1}\Gamma\left(c_2\right)\Omega_{02}^{c_2}}\cdot\sum_{j=1}^{N_1} p_{z_{1j}}\left(z_{1j}/\Omega_1\right)\prod_{\substack{k=1\\k\neq j}}^{N_1} F_{z_{1k}}\left(z_{1k}/\Omega_1\right) \quad (12)$$

$$H_2 = \frac{1}{\Gamma(c_1)\,\Omega_{01}^{c_1}\Gamma(c_2)\,\Omega_{02}^{c_2}} \cdot \sum_{j=1}^{N_2} p_{z_{2j}}(z_{2j}/\Omega_2) \prod_{\substack{k=1\\k\neq j}}^{N_2} F_{z_{2k}}(z_{2k}/\Omega_2) \tag{13}$$

After substituting (4), (10) in (2), and following simillar mathematical procedure, we obtain following expressions for normalized average fade duration, AFD:

$$\begin{aligned} T_z(z) &= \frac{\int_0^\infty \prod_{\substack{k=1\\k\neq j}}^{N_1} F_{z_{1k}}(z_{1k}/\Omega_1) \frac{1}{\Gamma(c_1)\Gamma(c_2)} \cdot \frac{\Omega_1^{c_1-1}}{\Omega_{01}^{c_1}} \cdot e^{-\frac{\Omega_1}{\Omega_{01}}} \cdot \gamma\left(\Omega_1/\Omega_{02}, c_2\right) d\Omega_1}{N_r(r)} + \\ &+ \frac{\int_0^\infty \prod_{\substack{k=1\\k\neq j}}^{N_2} F_{z_{2k}}(z_{2k}/\Omega_2) \frac{1}{\Gamma(c_1)\Gamma(c_2)} \cdot \frac{\Omega_2^{c_2-1}}{\Omega_{02}^{c_2}} \cdot e^{-\frac{\Omega_1}{\Omega_{01}}} \cdot \gamma\left(\Omega_2/\Omega_{01}, c_1\right) d\Omega_2}{N_r(r)} \end{aligned} \tag{14}$$

2.2 Maximal ratio combining at micro level

This macrodiversity system is of SC type and consists of two microdiversity systems with selection based on their output signal power values. Each microdiversity system is of MRC type with arbitrary number of branches in the presence of Rayleigh fading. Treating the correlation between the branches as exponential, the expression for the pdf of the SNR at the outputs of microdiversity systems follows [8]:

$$p(z_i/\Omega_i) = \frac{1}{\Gamma(M_i)} \left(\frac{N_i}{r_i\Omega_i}\right)^{M_i} z_i^{M_i-1} \exp\left(-\frac{N_i}{r_i\Omega_i} z\right) \tag{15}$$

In pervious equation, $\Gamma(x)$ denotes the Gamma function. N_i denotes the number of identically assumed channels at each microlevel. Number of diversity branches at the microlevel can be arbitrary. However, since channels are consider correlated, micro diversity system is applied on small terminals where spacing between the diversity branches is small [3]. Also there is no need to increase significantly number of diversity branches, because achieved output performance improvement with few diversity branches would not increase very much with the appliance of more branches. So the limitation for the number of diversity branches in the micro-diversity system is the trade-off between the complexity of practical realization and requested performance improvement. Parameter r_i is related to the exponential correlation $\rho_i$among the branches and is given with:

$$r_i = N_i + \frac{2\rho_i}{1-\rho_i}\left[N_i - \frac{1-\rho_i^{N_i}}{1-\rho_i}\right] \tag{16}$$

Parameter M_i is defined as:

$$M_i = \frac{N_i^2}{r_i} \tag{17}$$

Since the outputs of a MRC system and their derivatives follow [7]:

$$z_i^2 = \sum_{k=1}^{N_i} z_{ik}^2 \qquad \text{and} \qquad \dot{z}_i = \sum_{k=1}^{N_i} \frac{z_{ik}}{z_i} \dot{z}_{ik} \quad i = 1,2 \tag{18}$$

then $\dot{z}_i$ is a Gaussian random variable and with zero mean:

$$p(\dot{z}_i) = \frac{1}{\sqrt{2\pi}\dot{\sigma}_{z_i}} \cdot \exp\left(-\frac{\dot{z}_i^2}{2\dot{\sigma}_{z_i}^2}\right) \tag{19}$$

and variance given with [7]:

$$\dot{\sigma}_{z_i}^2 = \sum_{k=1}^{N_i} \frac{z_{ik}^2 \dot{\sigma}_{z_{ik}}^2}{z_i^2} \tag{20}$$

For the case of equivalently assumed channels, when stands: $\dot{\sigma}_{z_{i1}}^2 = \dot{\sigma}_{z_{i2}}^2 = \ldots \dot{\sigma}_{z_{ik}}^2$, $k = 1,\ldots,N$ pervious reduces into:

$$\dot{\sigma}_{z_i}^2 = \dot{\sigma}_{z_{ik}}^2 = \pi f_d^2 \Omega_i, \tag{21}$$

where f_d is a Doppler shift frequency.

Conditioned on Ω_i, the joint PDF $p(z_i, \dot{z}_i/\Omega_i)$ can be calculated as (10). It is already quoted that our macrodiversity system is of SC type and that selection based on the micro-combiners output signal power values. This selection can be written through the first order statistical parameters PDF and cumulative distribution function (CDF) at the macrodiversity output in the form of (3) and (4).

Since base stations at the macrodiversity level are widely located, due to sufficient spacing between antennas, signal powers at the outputs of the base stations are moddeled as statistically independent. Here long-term fading is described with Gamma distributions, which are as above mentioned independent, as:

After substituting (15), (19) and (21) into (1), and following the procedure explained in them we can easily derive the infinite-series expression for the system output LCR, in the form of:

$$\begin{aligned} \frac{N_Z(z)}{f_d} &= \left[\int_0^\infty W_1 \gamma\left(c_2, \frac{\Omega_1}{\Omega_{02}}\right) \cdot \Omega_1^{c_1-1-M_1-1/2} \cdot \exp\left(-\frac{\Omega_1}{\Omega_{01}} - \frac{N_1}{r_1}\frac{z}{\Omega_1}\right) \dot{\sigma}_z^2 \, d\Omega_1 + \right. \\ &+ \left. \int_0^\infty W_2 \gamma\left(c_1, \frac{\Omega_2}{\Omega_{01}}\right) \cdot \Omega_2^{c_2-1-M_2-1/2} \cdot \exp\left(-\frac{\Omega_2}{\Omega_{02}} - \frac{N_2}{r_2}\frac{z}{\Omega_2}\right) \dot{\sigma}_z^2 \, d\Omega_2 \right] \end{aligned} \tag{22}$$

with W_1 and W_2 given with:

$$W_1 = \frac{1}{\Gamma(M_1)} \left(\frac{N_1}{r_1}\right)^{M_1} \frac{z^{M_1-1}}{\Gamma(c_1)\Gamma(c_2)\Omega_{01}^{c_1}\sqrt{8\pi^3 z}} \tag{23}$$

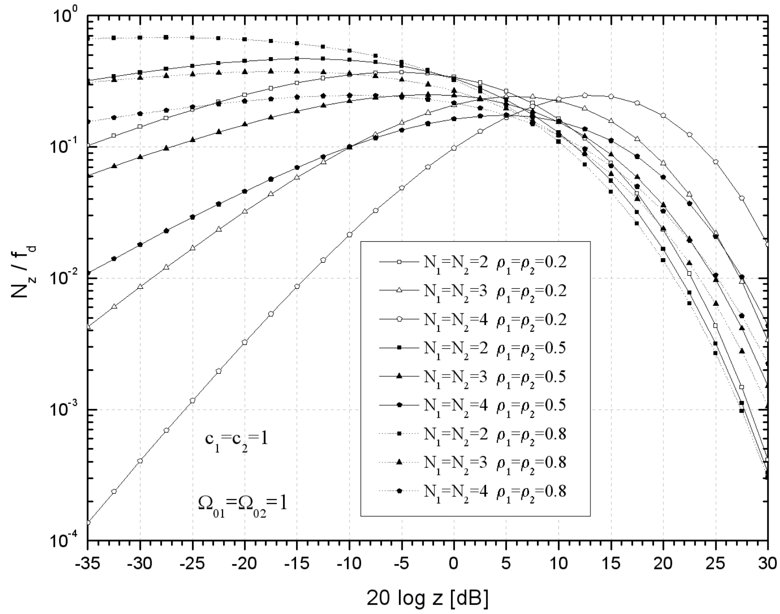

Fig. 1. Normalized average LCR of our macrodiversity structure for various values of correlation level and diversity order when MRC techniques are applied at microlevel

$$W_2 = \frac{1}{\Gamma(M_2)} \left(\frac{N_2}{r_2}\right)^{M_2} \frac{z^{M_2-1}}{\Gamma(c_1)\Gamma(c_2)\Omega_{02}^{c_2}\sqrt{8\pi^3 z}} \tag{24}$$

In the similar manner from (4) we can obtain an infinite series expression for the output AFD.

3 Numerical results

In order to show the influence of various parameters such as number of the diversity branches at the microcmbiners, fading severity and level of correlation between those branches, and type of diversity applied at micro-level on the system's statistics, numerical results are given and graphically presented. Normalized values of LCR, by maximal Doppler shift frequency f_d are presented at Figures 1 and 2. Numerical results for LCR are presented in the function of normalized signal level. Signal level is normalized with the sqare root of mean power of Gamma distributed signal , $r = z/\sqrt{\Omega_{01}}$.

We can observe from Fig. 1, which shows applience of MRC technique, that lower levels are crossed with the higher number of diversity branches at each microcombiner, and lower level of correlation between the branches.

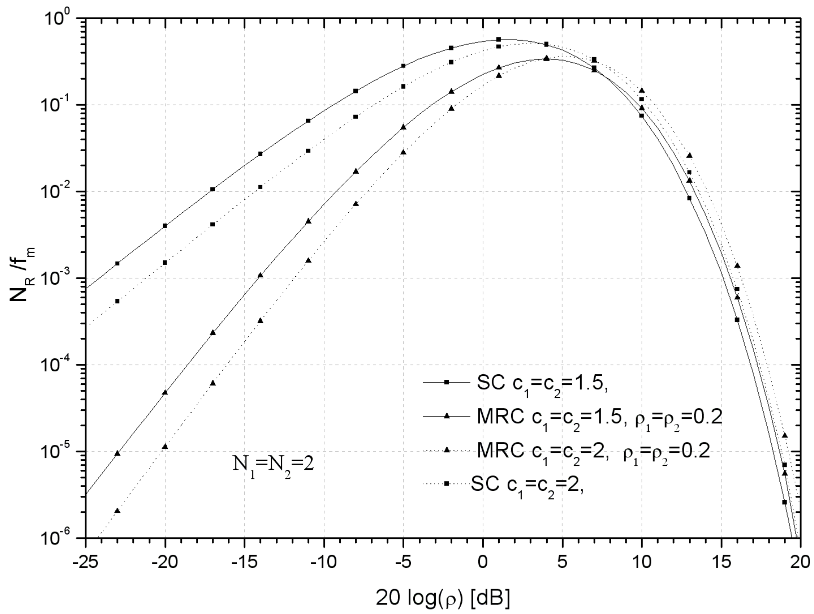

Fig. 2. Normalized average LCR of our macrodiversity structure for various values of correlation level and diversity order when SC and MRC techniques are applied at microlevel.

From Figure 2, which shows applience of SC technique, can be seen, that for the normalzed signal levels, which are $r < 0$ dB, has smaller values in observed range in the presence of shadowing with smaller values of parameters c_1i c_2. Finally, as we have expected, by comparing LCR values, conclusion can be driven, about better system performances for the case when MRC combining is used at each base station.

4 Conclusion

Performance analysis of macrodiversity combining system over Gama shadowed Rayleigh fading channels is presented in this paper. Analysis has been performed for the cases when both MRC and SC techniques are performed at each micro-level. Numerical results are graphically presented in order to show the influence of various system parameters on macrodiversity output LCR values.

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