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On Analytical Investigation and Computer Modeling in the Problem of the Satellite'S Attitude Stabilization

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Abstract: The satellite with electric charge and the proper magnetic moment is under consideration. The charge center position and and the proper magnetic moment are under control. The satellite's attitude control system based on the influence of the Lorentz torque and the torque of magnetic interaction with the Earth's magnetic field is used for the satellite's stabilization in the orbital frame. The asymptotic stability of stabilized satellite motion is proved. At the same time the accurate preliminary analysis of the forces acting upon a satellite and the construction of thoroughly analyzed mathematical model of the problem is the urgent task for achievement the required stabilization accuracy. Therefore the software complex was designed for comprehensive analysis of problems concerning the satellite attitude dynamics in geophysical fields. The paper present the results obtained analytically and by the computer modeling.

Keywords: Attitude control, geomagnetic field, Lorentz moment, electro-dynamical interaction, computer modeling.

1 The concept of electro-dynamical attitude control system

The paper deal with a satellite in a circular near-Earth orbit. The satellite is equipped with electrically charged shield with the total charge Q and possesses the proper magnetic moment $\vec{I}$. The charge center position, determined by vector $\vec{\rho}_0$ in the frame, rigidly connected with the satellite, and the proper magnetic moment $\vec{I}$ are under control. When the spacecraft is moving through the Earth's magnetic field with the magnetic induction $\vec{B}$ the interaction of the shield charge with the geomagnetic field results in Lorentz forces excitation. The principal moment of these forces with respect to the spacecraft mass center may be approximated by the formula [3]

$$\vec{M}_L = \vec{P} \times \vec{T}, \tag{1}$$

where $\vec{P} = Q\vec{\rho}_0$, $\vec{T} = \vec{v}_C \times \vec{B}$, $\vec{v}_C$ is the satellite's relative velocity with respect to the Earth's magnetic field. The value of vector $\vec{B}$ in these formulas coincide with the value of vector $\vec{B}$ in the satellite's mass center.

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The magnetic interaction between the satellite and the Earth's magnetic field causes the magnetic torque

$$\vec{M}_M = \vec{I} \times \vec{B}, \tag{2}$$

The mentioned torques of electro-dynamical interaction between the satellite and the Earth's magnetic field influences significantly on the satellite's attitude dynamics and can be used for the construction of the satellite attitude control systems. In the stated problem the Lorentz and magnetic torques are variable in accordance with the laws of control prescribed to vectors $\vec{P}$ and $\vec{I}$.

It is well known that the magnetic moment (2) may be used for the attitude stabilization of a satellite. Such magnetic control systems may be successfully used on the satellites working for a long time because they are rather simple, possesses high reliability and don't need for the consumption of some working material. But these systems have some specific features which restrict their opportunities. As it follows from (2), the magnetic moment is orthogonal to vector $\vec{B}$. So it is impossible to construct the control magnetic moment directed along vector $\vec{B}$.

The electrostatic attitude control system was proposed in [4] and developed in [2] and [7]. This system exploits the stabilizing effect of the Lorentz torque. The usage of this moment don't need the consumption of any working material, don't need to move any heavy bodies and exhibits such advantages as the simplicity of the control law, reliability, economical operation, little mass. Moreover, the main components of described control system may be used not only for attitude stabilization of a satellite, but also for its electrostatic radiation screening. The convenience of this method is proved analytically and verified by computer modeling. But, the specific feature of the systems, exploiting $\vec{M}_L$ for the satellite's stabilization is that vector $\vec{B}$ is orthogonal to vector $\vec{T}$. So it is impossible to construct the control Lorentz moment $\vec{M}_L$ directed along vector $\vec{T}$. Comparing the mentioned above specific features of magnetic and electrostatic systems and the following from them disadvantages of these systems, one can easily notice that these disadvantages may be avoided by constructing the united electro-dynamical attitude control system, simultaneously exploiting the moments $\vec{M}_L$ and $\vec{M}_M$. Indeed, the disadvantages rising from the existence of such directions along which is possible uncontrolled rotation of the satellite, vanishes as soon as we unite $\vec{M}_L$ and $\vec{M}_M$ in one electro-dynamical attitude control system. This fact can be explained by taking into consideration that vector $\vec{T}$ is orthogonal to vector $\vec{B}$ and consequently there are no such directions along which uncontrolled rotation of the satellite is possible.

2 From the concept to mathematical model

It is proved that under some conditions there exist control algorithms for the satellite electrodynamic parameters that ensures the satellite attitude stabilization. These algorithms were obtained in an explicit forms of functions depending on attitude variables with quasi-periodic coefficients. So the control algorithms for the satellite electromagnetic parameters, which allows to stabilize the satellite attitude position in the orbital frame are obtained. The method of electrodynamic attitude control was also successfully used for the problems of one-axis satellite stabilization and for the stabilization of a satellite rotation in the orbital coordinate system. The asymptotic stability of stabilized satellite motion is proved analytically [1], [2], [4]. At the same time the accurate preliminary analysis of the forces acting upon a satellite and the construction of thoroughly analyzed mathematical model of the problem is the urgent task for achievement the required stabilization accuracy.

Therefore the software complex was designed for comprehensive analysis of problems concerning the satellite attitude dynamics in geophysical fields. Through the use of computer algebra methods the problem of constructing the simplest possible, but correct mathematical model that ensures the prescribed accuracy is solved. As a result the octupole approximation of the Earth's magnetic field was accepted and the stability of the satellite's stabilized orientation was proved analytically and confirmed by computer modeling. The report describes the complex as a complete software product, designed primarily to assess the effectiveness of the method of electrodynamic stabilization of satellites, combined with the possibility of using some of its modules to address a wide variety of local problems.

One of these local problems is the problem of choice rather simple and correct mathematical model of the Earth's magnetic field which ensures the prescribed accuracy. Unfortunately, we can not confine ourselves to simple models that are usually used in the preliminary analytical analysis of the problem, and the refinement of models does not allow to keep the task within the capabilities of man. For example, to take into account the impact of the Earth's magnetic field on the dynamics of the satellite, we need to know the induction vector $\vec{B}$ as a function of the radius-vector of the point of near-Earth space. However, in reality, the Earth's magnetic field have a very complicated structure, so that this functional dependence is absent in the analytical form. Hence the need for mathematical modeling of the Earth's magnetic field in those tasks in which accuracy requirements does not allow us to restrict ourselves to the simplest well-known approximations (such as dipole magnetic field). The developed set of programs aimed at solving these problems by using computer algebra capabilities implemented in Maple, and further use of the constructed models for the numerical analysis.

3 Analytical and numerical results

For the first time the algorithm for analytical multipole representation of the Earth's magnetic field with an arbitrary degree of accuracy was constructed. The mathematical justification of this method was given in [3] and developed in [6]. The algorithm and software for Maple, implements symbolic computation, allowing to build the analytical expressions of multipole tensors of arbitrary rank, expressing them in terms of Gauss coefficients, is developed. A program that allows to obtain vector $\vec{B} = \sum_{n=1}^{N} \vec{B}^{(n)}$ in any finite approximation is constructed. The near-Earth space was divided into regions in which the account of a finite number of vector $\vec{B}$ multipole components is correct due to the chosen criteria of accuracy. This allows us to establish the necessary and sufficient number of terms in the multipole expansion of vector $\vec{B}$, which provides the required accuracy of finding this vector at any point in the satellite orbit with the given radius and inclination. A program for evaluation the system of differential equations in Rodrigues-Hamilton parameters describing the satellite's attitude motion is constructed. A program for numerical solution of the Cauchy problem in the satellite electrodynamic stabilization and further numerical analysis of this solution and its visualization is written. In addition, the complex contains a number of software modules to automate the following procedures: the linearization of differential equations, the finding the stability regions for the solutions of differential equations in the parameters planes, the optimization of the electrodynamic control [7].

Let us illustrate some results which one can obtain with the help of designed software complex. The components of the first seven multipole tensors are obtained analytically. Thus, the tensor of the 7-th rank have $3^7 = 2187$ components, but only 36 of them are different due to the property of symmetry with respect to any pair of indices. For example we represent only three of these components:

$$M_{2233333} = -\frac{1}{14}\left(\sqrt{42}g_7^2 + 7\,g_7^0\right), \quad M_{2333333} = \frac{2\sqrt{7}}{7}\,h_7^1, \quad M_{3333333} = g_7^0.$$

Another result one can see in Fig. 1 where the horizontal axes correspond to the inclination of the orbit and the orbit's radius divided by the Earth's radius, and the vertical axis correspond to the norms of the multipole components of geomagnetic induction $||\vec{B}^{(n)}||$ $(n \geqslant 2)$ averaged for the orbital motion and divided by the averaged norm of dipole component $||\vec{B}^{(1)}||$ of geomagnetic induction. The horizontal sections of obtained surfaces allows to find uniquely the necessary and sufficient number of terms in the Earth's magnetic induction expansion which provides the prescribed precision. The section corresponding to 10% accuracy one can see in Fig. 1.

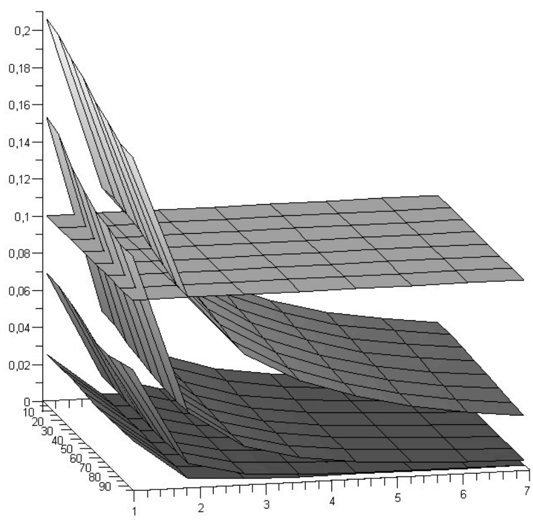

Fig. 1.

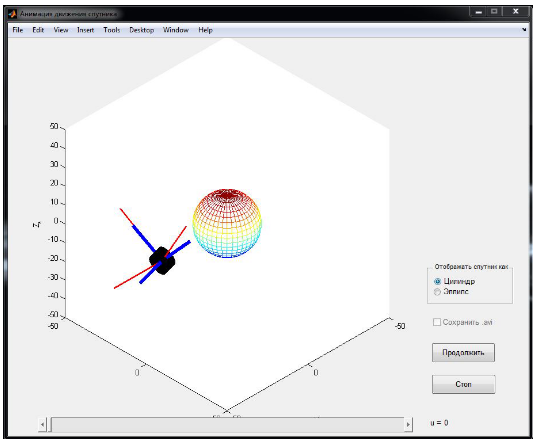

Fig. 2.

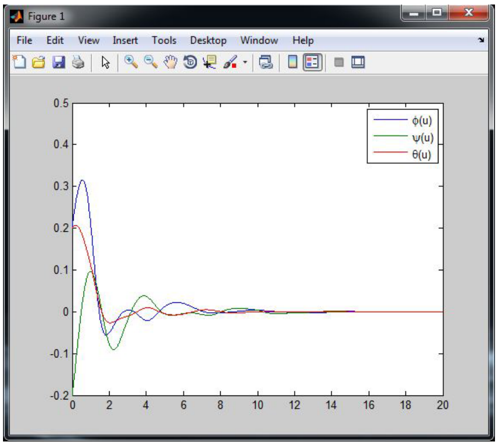

Fig. 3.

The designed software complex allows to watch the 3D-animation of the process of satellite's electrodynamic attitude stabilization. The fragment of this animation is shown in Fig. 2. Thin red axes correspond to the orbital frame and thick blue axes correspond to the satellite's principal central axes of inertia.

As a result of integration of differential equations of the satellite's attitude motion one can see the time history graphics of Rodrigues-Hamilton parameters, the angles of orientation of the satellite (shown in Fig. 3), and the magnitudes of control and disturbing moments.

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