SCIENTIFIC PUBLICATIONS OF THE STATE UNIVERSITY OF NOVI PAZAR
SER. A: APPL. MATH. INFORM. AND MECH. vol. 4, 2 2012, 55-64

Sliding Mode Control Based on Orthogonal Models

D. Antić, M. Milojković, D. Mitić, S. Nikolić, S. sa Perić

Abstract: This paper presents a new method for the design of sliding mode control. Sliding mode control is a robust control method convenient to use with the systems with nonlinear and uncertain characteristics. Design is based on the linearized models of dynamical systems obtained using novel forms of orthogonal functions and filters developed specifically for the analysis of technical systems. Good performances of the designed controller are veri?ed by performed real time experiments on the laboratory experimental setup of an antilock braking system.

Keywords: Sliding mode control, linearized model, orthogonal functions, antilock braking system

1 Introduction

Theory of orthogonal functions is constantly developing in the last few centuries [6]. Today we have many applications of orthogonal polynomials in many technical fields: designing orthogonal filters, signal generators, process modeling and identification, practical realizations of optimal and adaptive systems. New methods for obtaining orthogonal rational functions and their applications in the field of engineering have been the subject of intensive research of authors of this paper. In [2], [9] we present a new method for designing orthogonal rational functions using specific transformations in complex domain and derive a necessary mathematical background. We also present a new method for obtaining models of continuous dynamical systems based on these orthogonal functions [7]. Practical realization of derived orthogonal filters is very simple and they are very fast, robust and precise. They are also very convenient for application of gradient methods in optimization and adaptation problems because of their feature to speed up existing (classical) control algorithms. In this paper we use developed Muntz-Legendre orthogonal filter to improve existing sliding mode control technique by building linearized models of the plant to be controlled.

Sliding mode control [12], [14] is a popular approach to the robust control of uncertain systems. The principal goal of the sliding mode control technique is to force a system

Manuscript received March 17, 2012 ; accepted May 27, 2012.
Dragan Antić, Marko Milojković, Darko Mitić, Saša Nikolić, Staniša Perić are with University of Niš, the Faculty of Electronic Engineering, Niš, Serbia.

state to a certain prescribed manifold, known as the sliding surface. Once the manifold is reached, the system is forced to remain on it thereafter. When in the sliding mode, the system is equivalent to an unforced system of lower order, which is insensitive to both parametric uncertainty and unknown disturbances that satisfy the matching condition. The main drawback of the sliding mode control is the requirement of a discontinuous control law across the sliding manifold. In practical systems this leads to a phenomenon termed chattering. Chattering involves high-frequency control switching and may lead to excitation of unmodeled high-frequency system dynamics. Chattering also cause high heat losses in electronic systems and undue wear in mechanical systems.

In this paper, novel control algorithm will be tested on the problem of control of an antilock braking system (ABS). ABS is an electronic system helping secure and controlled abruptly stopping of the vehicle. Nowadays it represents the standard equipment of the vast majority of modern vehicles. The basic idea of ABS is to prevent the wheels to stop completely during a sudden braking. If that happens, the control over vehicle is lost and it can skid in an undesirable direction. ABS does not allow the wheels to be stiffened and thus enable driver to normally operate with the vehicle, although the brake pedal is pressed to the end. ABS mathematical model is not fully developed due to its nonlinear structure, although this system has been used for several decades. System nonlinearities are reflected in unknown parameters of vehicle environment and nonlinear characteristics of braking dynamics. Besides that, the system parameters vary, which is caused by components deterioration, and many external disturbances cannot be predicted in advance. That is why sliding mode control (SMC) methods seem to be the right choice in the control of ABS, especially when combined and improved with orthogonal models.

2 Orthogonal models

In this paper, we use previously designed adjustable Müntz-Legendre orthogonal filter [2] shown in Fig. 1 for the purpose of the modeling of dynamical system, as a first step in designing the adequate control.

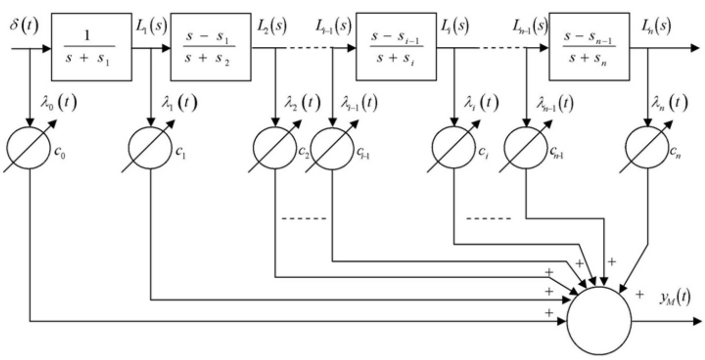

Fig. 1. Adjustable Müntz-Legendre orthogonal model

As already proven in [3], this filter generates orthogonal signals (λ_i) suitable for various applications in analysis and synthesis of technical systems. The following approximation is used to obtain the model of an arbitrary dynamical system:

$$y_M \approx \sum_{i=0}^{n} c_i \lambda_i(t). \tag{1}$$

During the modeling of concrete unknown system, parameters c_i should be adjusted in such a way that the model in Fig. 1 corresponds to the unknown system as exactly as possible. The process of modeling begins with applying the same input signal to both the unknown system and the adjustable model in Fig. 1. Next, the difference of system and model outputs is formed as well as mean square error:

$$J = \frac{1}{T}\int_0^T (y_S - y_M)^2 \, dt, \tag{2}$$

where $y_S(t)$ and $y_M(t)$ are systems' and models' output, respectively. Parameters c_i are being adjusted until the minimization of the function (2) is achieved, i.e., as long as necessary to obtain the best model of unknown system in the sense of mean square error. The complete scheme, that illustrates the process of modeling, is given in Fig. 2

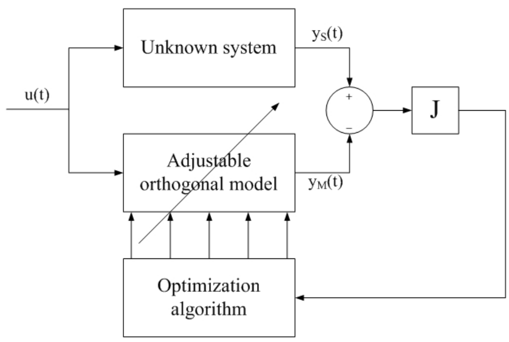

Fig. 2. Block diagram of the modeling process

3 Case study

Theoretical algorithms developed in this paper are verified on a case study of an anti-lock braking system (ABS). ABS is an electronic device, situated inside the vehicle control panel, which is activated when the possibility of blocking wheel is met. The main idea is that, during suddenly braking, the wheels do not block completely. The coefficient which characterizes the adhesion between the wheels and the road surface is known as road adhesion coefficient μ. It is defined as the proportion between the friction force and the normal load of the vehicle. This coefficient is in nonlinear dependence on the wheel slip λ, defined as the relative speed difference between the wheel and vehicle. Most of the controllers are designed to regulate wheel slip on the pre-set level, in desired range, so that the road adhesion coefficient has its maximal value for that level of wheel slip. The ABS dynamic has strong nonlinear nature, so a robust control method (like sliding mode control used in this paper) tends to be a logical choice.

The ABS experimental setup, shown in Fig. 3, consists of two wheels which are permanently in rolling contact. While the upper wheel is equipped with a tire, representing the vehicle wheel, the lower one, representing the relative road motion, has smooth surface. The upper wheel is equipped in the disk brake system connected via hydraulic coupling to the brake lever which by the tight side and tightening pulley is driven by the small dc motor. The steel cord causes strong nonlinearity and limitation of control input signal on 50% of its maximum nominal value. The lower wheel is coupled to the big flat dc motor whose task is to accelerate the wheel. During the braking phase its power supply is switched off.

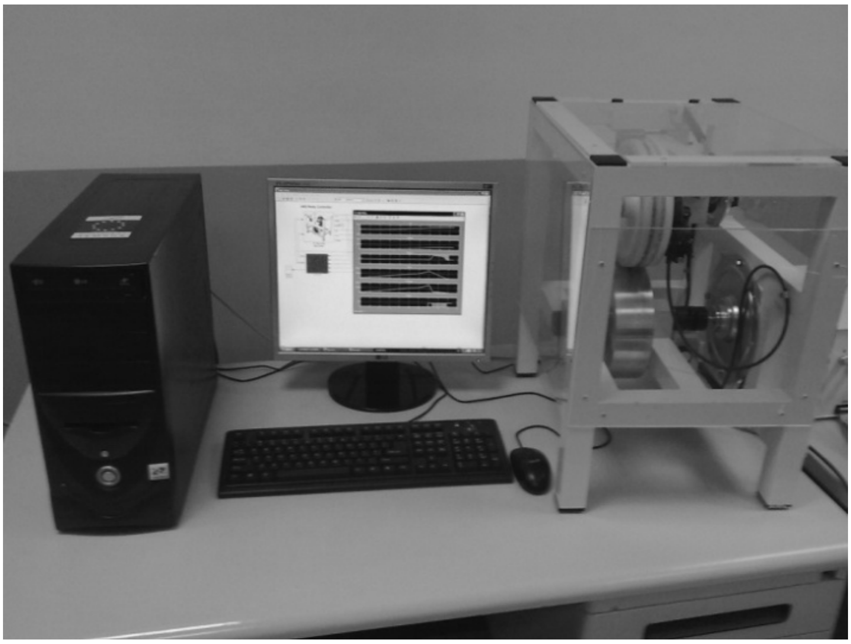

Fig. 3. ABS laboratory setup

4 Sliding mode control

Sliding mode control (SMC) belongs to the well-studied class of discontinuous control [4], [5], [13] methods. Sliding mode is of particular interest in nonlinear systems and it occurs when the system state is forced to move along a predefined sliding surface, determined by the so-called switching function. If sliding mode exists, a system becomes robust to parameter variations and external disturbances, and its dynamics is known in advance and usually of the low order. The shortcoming of SMC is the existence of the chattering phenomenon. It occurs as the consequence of the high frequency control signal, which can excite the system unmodelled dynamics.

All sliding mode control algorithms described in this paper are based on the wheel slip dynamics and consist of two terms: the relay component ensuring the system state to reach the sliding surface from any initial point and the so-called equivalent control, which keeps the system state on it. The first algorithm is the traditional SMC, representing the simpler form of this robust control [10]. Since the system is of the first order, the switching function is selected as:

$$\sigma = \lambda - \lambda_r, \tag{3}$$

where λ_r is the reference wheel slip. The main control design objective is to find control providing $\sigma = 0$ and consequently $\lambda = \lambda_r$. For the system motion in sliding mode $\dot{\sigma} = 0$, the equivalent control is calculated according to:

$$\dot{\sigma} = 0 \Rightarrow \dot{\lambda} = 0 \Leftrightarrow f(\lambda, x_2) + g(\lambda, x_2) M_1^{eq} = 0, \tag{4}$$

under the assumption that λ_r=const. Equation (3) yields the equivalent control brake torque in the following form:

$$M_1^{eq} = -g(\lambda, x_2)^{-1} f(\lambda, x_2). \tag{5}$$

The traditional SMC brake torque is then determined as:

$$M_1 = M_1^{eq} - \bar{M}_1 sgn(\sigma), \tag{6}$$

where the parameter $\bar{M}_1$ is chosen to satisfy the reaching and existence condition of sliding mode:

$$\sigma\dot{\sigma} < -\eta\,|\sigma|, \tag{7}$$

To improve the system accuracy, we can use the second algorithm with the integral switching function [1]:

$$\sigma = (\lambda - \lambda_r) + c_1 \int (\lambda - \lambda_r)\, dt, \tag{8}$$

This control algorithm may have influence on chattering reduction as well. As in the previous case, the equivalent control brake torque is determined from:

$$\dot{\sigma} = 0 \Rightarrow f(\lambda, x_2) + g(\lambda, x_2) M_1^{eq} + c_1(\lambda - \lambda_r) = 0, \tag{9}$$

resulting in:

$$M_1^{eq} = -g(\lambda, x_2)^{-1} \left(f(\lambda, x_2) + c_1(\lambda - \lambda_r) \right). \tag{10}$$

5 Sliding mode control design using orthogonal model

Application of orthogonal functions in ABS control is based on linearization of the system in the working areas. These simplified models don't represent full complexity of the real system but they determine systems' dynamics in specific conditions adequately enough. Paper [11] demonstrates that ABS linearized model can be represented with the following transfer function (input - applied brake torque, output - wheel slip):

$$W(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0} \tag{11}$$

The idea proposed in this paper is to record step responses of the system (values of wheel slip λ) in the nine working areas, for various levels of applied control (braking force) evenly selected from the domain of possible values [0-0.4] [8]. Based on these responses we can linearize system in the given areas using orthogonal polynomials, as already described in Section 2. For this case, we use adjustable Müntz-Legendre orthogonal filter shown in Fig. 1 with two sections, which results in the desired form of transfer function (11). The obtained results are shown in Table 1.

Table 1. PARAMETERS OF THE LINEARIZED SYSTEM FOR DIFFERENT WORKING AREAS

working area	a_1	a_0	b_1	b_0
1	0.4870	38.5466	64.631	935.3293
2	0.1385	73.9746	92.113	1409.4
3	0.7042	17.922	31.129	242.0561
4	0.9414	5.7735	44.013	-29.4233
5	0.9966	20.9434	43.476	-115.7159
6	0.6324	4.7556	1.401	-7.3129
7	0.6230	10.1635	3.638	-13.806
8	0.7194	10.7825	1.183	-3.0782
9	0.3052	21.9169	3.482	2.3701

To design the controller using the proposed method, we use system model in canonical form:

$$\begin{aligned} \dot{x}_1 &= d_{11}x_1 + d_{12}x_2 + ku \ , \\ \dot{x}_2 &= d_{21}x_1 + d_{22}x_2 \ , \\ y &= x_1 \ . \end{aligned} \tag{12}$$

where d_{11}=$(b_0 - a_1 b_1)/b_1$, $d_{12} = b_0(b_0 - a_1 b_1)\ /b_1 - a_0 b_1$, d_{21}=$1/b_1$, d_{22}=$-b_0/b_1$, $k = b_1$.

For a given form of the system, the sliding mode control can be designed as [14]:

$$u = u_{eq} + \alpha sgn\left(x_1^{ref} - x_1\right), \tag{13}$$

where x_1 represents current slip value λ, with λ_{ref} chosen to be 0.2. Equivalent control can be obtained from the condition of the sliding mode existence.

$$u_{eq} = -\frac{d_{11}x_1}{k}. \tag{14}$$

6 Experimental results

The experimental results obtained by all three control methods (classical sliding mode control - CSMC, integral sliding mode control - ISMC and orthogonal sliding mode control - OSMC), described in chapters 4 and 5, are shown in Figures 4, 5 and 6.

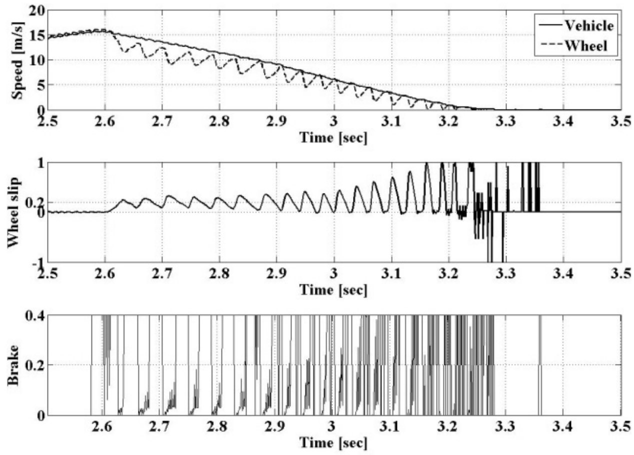

Fig. 4. Classical sliding mode control

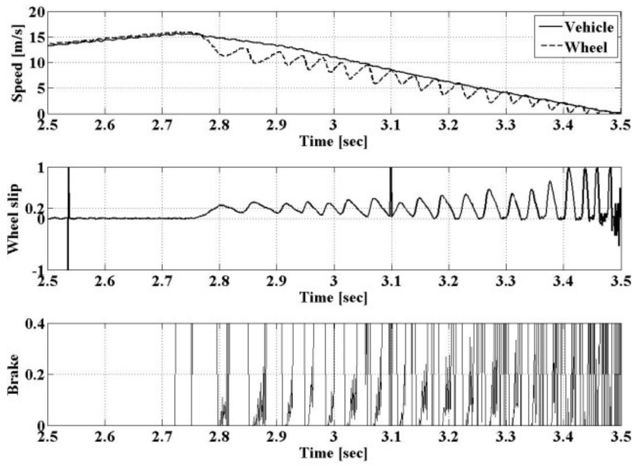

Fig. 5. Integral sliding mode control

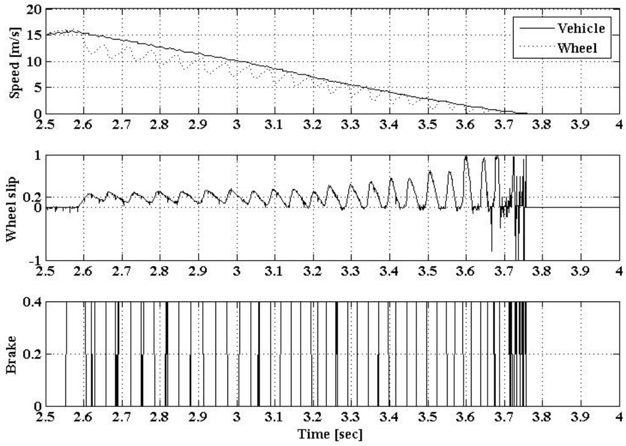

Fig. 6. Orthogonal sliding mode control

Comparative analysis of the results of all described control methods is given in Table 2. In this table we use three parameters as relevant for evaluation of the quality of controllers. The first parameter is T_z and it represents stopping time, i.e., duration from the beginning of braking to a complete halt of the vehicle. The second parameter is the mean squared error, which is calculated by the formula $E = \int_0^{T_z} (\lambda - \lambda_{ref})^2 \, dt$. The third parameter is N - the total number of changes in levels of control during the braking process reflecting wear of ABS system during working. It is obviously desirable for each of the control methods that all three parameters are as low as possible.

Table 2. COMPARATIVE ANALYSIS OF DIFFERENT CONTROL METHODS

Control method	T_z	E	N	J
CSMC	1.03	48.6750	230	4.5910
ISMC	1.14	45.0433	254	4.7965
OSMC	1.07	32.0758	69	3.4237

The table shows that the classical and integral sliding mode controls give similar results. Using the orthogonal models with the sliding mode control improves performance of controlled system significantly. This can be best seen if we introduce a comprehensive performance index (J in Table 2), which combines all three, described above, relevant parameters for the analysis of the quality of designed controllers. This index is obtained by a simple formula: $J = k_1 T_z + k_2 E + k_3 N$ where the coefficients k_1, k_2, and k_3 can be adjusted to emphasize the importance of each of the three relevant parameters on the overall performance index. The values in column J of the Table 2, were obtained by selecting k_1=1.95504, k_2=0.03197 and k_3=0.00444. These coefficients provide the normalization of the parameters with highlighting the importance of the stopping time of vehicles (the most important parameter for safety) by doubling the normalized values. The obtained values of the index confirm the quality of the performance of designed sliding mode control based on orthogonal functions and models.

7 Conclusion

This paper presented a new method for the design of the sliding mode control, a robust control method convenient to use with the systems with nonlinear and uncertain characteristics. Control is improved by using specific orthogonal filter already designed by authors. This filter is based on the theory of orthogonal rational functions (derived via specific transformations in complex domain) and their applications in the field of engineering. In this paper, filter is used for obtaining linearized models of systems to be controlled. Performances of the designed controller are tested by performed real time experiments on the laboratory experimental setup of an antilock braking system. ABS is a perfect case study for testing our sliding mode control method because of its nature, uncertain and varying parameters. Experiments and comparative analysis with two simpler sliding mode controllers proved good performances of designed controller in the sense of stopping time (braking distance), achieved wheel slip and system wear. Of course, improved performances come with more complex and more expensive system.

Acknowledgment

This paper was realized as a part of the projects III 43007, III 44006 and TR 35005, financed by the Ministry of Education and Science of the Republic of Serbia.

References

[1] ANTIĆ, D., V. NIKOLIĆ, D. MITI, M. MILOJKOVIĆ, S. PERIĆ, *Sliding mode control of anti-lock braking system: An overview*, Facta Universitatis Series: Automatic Control and Robotics, Vol. 9, 1 (2010), 41-58.

[2] ANTIĆ, D., B. DANKOVIĆ, S. NIKOLIĆ, M. MILOJKOVIĆ, Z. JOVANOVIĆ, *Approximation based on orthogonal and almost orthogonal functions*, Journal of the Franklin Institute, Vol. 349, 1 (2012), 323-336.

[3] DANKOVIĆ, B., D. ANTIĆ, Z. JOVANOVIĆ, S. NIKOLIĆ, M. MILOJKOVIĆ, *Systems Modeling Based on Legendre Polynomials*, Proceedings of the 5th International Symposium on Applied Computational Intelligence and Informatics, SACI 2009, Timisoara, Romania, May 28.-29., 2009, pp. 241-246.

[4] GAO, W., J. C. HUNG, Variable structure control of nonlinear systems: A new approach, IEEE Transactions on Industrial Electronics, Vol. 40, 1 (1993), 45-55.

[5] HUNG, Y., W. GAO, J.C. HUNG, *Variable structure control: A survey*, IEEE Transactions on Industrial Electronics, Vol. 40, 1 (1993).

[6] KLINE, M., Mathematical Thought from Ancient to Modern Times, Oxford University Press, New York, 1972.

[7] MILOJKOVIĆ, M., S. NIKOLIĆ, B. DANKOVIĆ, D. ANTIĆ, Z. JOVANOVIĆ, *Modelling of dynamical systems based on almost orthogonal polynomials*, Mathematical and Computer Modelling of Dynamical Systems, Vol. 16, 2 (2010), 133-144.

[8] MILOJKOVIĆ, M., Orthogonal Functions Applied in Modeling of Dynamical Systems and Control of Systems with Fuzzy Sliding Mode, *PhD Dissertation*, Faculty of Electronic Engineering, Niš, 2012.

[9] NIKOLIĆ, S., D. ANTIĆ, B. DANKOVIĆ, M. MILOJKOVIĆ, Z. JOVANOVIĆ, S. PERIĆ, *Orthogonal functions applied in antenna positioning*, Advances in Electrical and Computer Engineering, Vol. 10, 4 (2010), 35-42.

[10] ONIZ, Y., E. KAYACAN, O. KAYNAK, *Simulated and experimental study of antilock braking system using grey sliding mode control*, IEEE International Conference on Systems, Man and Cybernetics, ISIC 2007, October 7.-10., 2007, pp. 90-95.

[11] PRECUP, R.-E., S. PREITL, B.M. RDAC, E.M. PETRIU, C.A. DRAGO, J.K. TAR, *Experiment-based teaching in advanced control engineering*, IEEE Transactions on Education, Vol. 54, 3 (2011), 345-355.

[12] UTKIN, V., *Sliding mode control design principles and application to electric drives*, IEEE Transactions on Industrial Electronics, Vol. 40, 1 (1993), 23-36.

[13] UTKIN, V., J. GULDNER, J. SHI, Sliding Mode Control in Electromechanical Systems, Taylor & Francis, 1999.

[14] YOUNG, K.D., V. UTKIN, U. OZGUNER, *A control engineer's guide to sliding mode control*, IEEE Transactions on Control Systems Technology, Vol. 7, 3 (1999), 328–342.